Int J Theor Phys (2008) 47: 1709–1712
DOI 10.1007/s10773-007-9612-y

Writing Quantum Data into One-Way Quantum Computer by Fusion Gate

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Received: 17 September 2007 / Accepted: 7 November 2007 / Published online: 22 November 2007

Abstract A scheme to write the quantum data into the one-way quantum computer by the type-I and type-II gates is given in this paper. It may provide a method to make full use of the resources including time and entanglement during quantum computation.

Keywords One-way quantum computer · Cluster state · Fusion gate

One-way quantum computation [1–4] formulate quantum computation in a way different from the conventional circuit model [5–7]. In this new model of quantum computation the quantum correlation in an entangled state named a cluster state [8] (or graph state [9]) is exploited to enable universal quantum computation through single-qubit projective measurement alone. The word "one-way" means that the cluster state can be used only once because the entanglement in it is destroyed by the one-qubit measurements. The cluster state is central physical resource of one-way quantum computation. A great deal of effort has been devoted to the effective and scalable generation of cluster state in various systems, such as optical lattices [10, 11], quantum dots [12, 13], linear optics [14–18] and cavity QED [15–19].

Cluster states can be created effectively in any system with a quantum Ising-type next neighbor interaction [1, 8]. For example, a cluster state for realizing an arbitrary rotation on single qubit $|\psi_{in}\rangle = \alpha|0\rangle + \beta|1\rangle$ can be prepared by the following two-stage procedure:

1. Initialize all qubits arranged in a 1D lattice, except for an quantum data qubit in the state $|\psi\rangle_{in}$, in the state $|+\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$, an eigenstate of the Pauli operator σ_x, where $|0\rangle$ and $|1\rangle$ are the eigenstates of the Pauli operator σ_z. The common state reads

$$|\Theta\rangle = |\psi_{in}\rangle_1 \otimes |+\rangle_2 \otimes |+\rangle_3 \otimes |+\rangle_4 \cdots \otimes |+\rangle_N.$$

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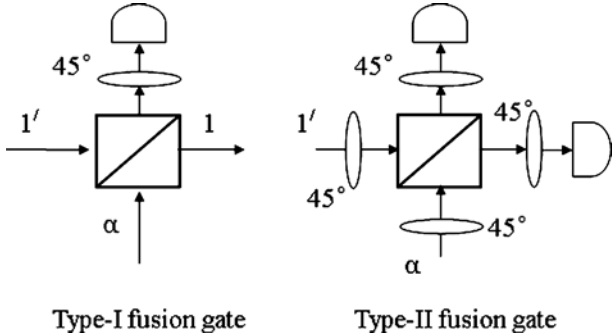

Fig. 1 The type-I and type-II fusion gates consist of polarizing beam splitters (PBS), 45° polarization rotation plates and polarization discriminating photon counters

2. Apply controlled-PHASE gates between neighbor qubits. Then an entangled cluster

$$|\Phi\rangle_{C_N} = \frac{1}{2^{(N-1)/2}}(\alpha|0\rangle_1\sigma_z^2 + \beta|1\rangle_1)(|0\rangle_2\sigma_z^3 + |1\rangle_2)\cdots(|0\rangle_N + |1\rangle_N)$$

is built.

Recently, Browne and Rudolph [14] has proposed an cheap way to grow cluster state effectively from Bell states by linear optics. The scheme they proposed used the probabilistic parity gates, and they referred to these gates as the type-I and type-II fusion gates which are illustrated in Fig. 1. Here, we find that the fusion gates is useful to load quantum information of unknown quantum state into a standard cluster state. If regarding the standard cluster state as quantum state of a one-way quantum computer, the unknown quantum state as data state, the loading process can be explained as writing the data into the quantum computer.

We first consider writing the quantum information of a single qubit a into a standard cluster by the type-I fusion gate. The qubit a is in the unknown quantum state $|\psi_{in}\rangle_a = \alpha|0\rangle + \beta|1\rangle$, the standard cluster state reads

$$|\Psi\rangle_{C_N} = \frac{1}{2^{N/2}}(|0\rangle_{1'}\sigma_z^2 + |1\rangle_{1'})(|0\rangle_2\sigma_z^3 + |1\rangle_2)\cdots(|0\rangle_N + |1\rangle_N).$$

For concreteness, we phrase this writing process in terms of qubits encoded in horizontal and vertical photon polarization, $|0\rangle$ denotes the polarization state $|H\rangle$ of photon, the state $|1\rangle$ denotes $|V\rangle$. The qubits a and $1'$ run into the fusion gate along two input modes of the PBS, and mix on the PBS. The qubit in one output mode undergoes a 45° polarization rotation before being detected by a polarization discriminating photon counter, the qubit in the other mode is labeled by 1. If one and only one H or V polarized photon is detected, the residual qubits are in a new cluster state:

$$\begin{aligned} \text{“}H\text{”}: |\Phi\rangle_{C_N} &= \frac{1}{2^{(N-1)/2}}(\alpha|0\rangle_1\sigma_z^2 + \beta|1\rangle_1)(|0\rangle_2\sigma_z^3 + |1\rangle_2)\cdots(|0\rangle_N + |1\rangle_N), \\ \text{“}V\text{”}: |\Phi\rangle'_{C_N} &= \frac{1}{2^{(N-1)/2}}(\alpha|0\rangle_1\sigma_z^2 - \beta|1\rangle_1)(|0\rangle_2\sigma_z^3 + |1\rangle_2)\cdots(|0\rangle_N + |1\rangle_N). \end{aligned} \tag{1}$$

For the case of a H polarized photon be detected, the new cluster state is just the cluster state built for realizing an arbitrary rotation on the qubit $|\psi_{in}\rangle$. For the other case of a V polarized photon be detected, an extra rotation σ_z on the qubit 1 can transform the cluster state $|\Phi\rangle'_C$ to $|\Phi\rangle_C$. If zero or two photons of either polarization are detected, the fusion operation fails, which happens with probability $\frac{1}{2}$. The failure gate operation has the effect of measuring the qubits a and $1'$ in the σ_z eigenbasis. The quantum state $|\psi_{in}\rangle_a$ is destroyed, and the qubit $1'$ is removed from the cluster state $|\Psi\rangle_C$. Another copy of the quantum state $|\psi_{in}\rangle_a$ is necessary to reattempt the fusion gate on the qubit a and the qubit 2 of the remaining cluster state

$$|\Psi\rangle_{C_{N-1}} = \frac{1}{2^{(N-1)/2}}(|0\rangle_2\sigma_z^3 + |1\rangle_2)\cdots(|0\rangle_N + |1\rangle_N).$$

The type-II fusion gate can also be used to writing the quantum information of a single qubit a into a standard cluster. Firstly, a σ_x measurement is performed on the qubit 2 in the cluster state $|\Psi\rangle_C$, which causes the neighboring qubits $1'$ and 3 to be joined into a single logical qubit in the redundant encoding, and the cluster state writes

$$|\Psi\rangle_{C_{N-1}} = \frac{1}{2^{(N-2)/2}}(|0\rangle_{1'}|0\rangle_3\sigma_z^4 + |1\rangle_{1'}|1\rangle_3)\cdots(|0\rangle_N + |1\rangle_N).$$

Then the qubits a and $1'$ are put into the type-II fusion gate. If each output mode detect a photon, the fusion operation succeeds. The resulting cluster state reads

$$\begin{aligned} \text{“}HH, VV\text{”}: |\Phi\rangle_{C_{N-2}} &= \frac{1}{2^{(N-2)/2}}(\alpha|0\rangle_3\sigma_z^4 + \beta|1\rangle_3)(|0\rangle_4\sigma_z^5 + |1\rangle_4)\cdots(|0\rangle_N + |1\rangle_N), \\ \text{“}HV, VH\text{”}: |\Phi\rangle'_{C_{N-2}} &= \frac{1}{2^{(N-2)/2}}(\beta|0\rangle_3\sigma_z^4 + \alpha|1\rangle_3)(|0\rangle_4\sigma_z^5 + |1\rangle_4)\cdots(|0\rangle_N + |1\rangle_N). \end{aligned} \tag{2}$$

For the case of the same polarized photon be detected in both output modes, the resulting cluster state is just the cluster state we need. For the other case of the different polarized photon be detected, the extra rotations σ_x on the qubit 3 and σ_z on the qubit 4 can transform the cluster state $|\Phi\rangle'_{C_{N-2}}$ to $|\Phi\rangle_{C_{N-2}}$. If zero photon in either output mode are detected, the fusion operation fails, which happens with probability $\frac{1}{2}$. The failure gate operation destroys the quantum state $|\psi_{in}\rangle_a$, and removes the qubit $1'$ from the cluster state $|\Psi\rangle_{C_{N-1}}$. Another copy of the quantum state $|\psi_{in}\rangle_a$ is necessary to reattempt the fusion gate.

The fusion operation of the type-I and type-II gates not only can be used to write quantum information into the terminal qubit of the standard cluster, but also into any “inner” qubit. For instance, running the qubit a and the qubit l of the standard cluster state

$$|\Psi\rangle_{C_N} = \frac{1}{2^{N/2}}(|0\rangle_1\sigma_z^2 + |1\rangle_1)\cdots(|0\rangle_{l-1}\sigma_z^l + |1\rangle_{l-1})(|0\rangle_l\sigma_z^{l+1} + |1\rangle_l)\cdots(|0\rangle_N + |1\rangle_N)$$

into the type-I gate will generate with probability $\frac{1}{2}$ a cluster state

$$\begin{aligned} |\Psi\rangle_{C_N} = \frac{1}{2^{(N-1)/2}}&(|0\rangle_1\sigma_z^2 + |1\rangle_1)\cdots(|0\rangle_{l-1}\sigma_z^l + |1\rangle_{l-1}) \\ &\times(\alpha|0\rangle_l\sigma_z^{l+1} + \beta|1\rangle_l)\cdots(|0\rangle_N + |1\rangle_N). \end{aligned}$$

But the failure type-I fusion operation will sever the standard cluster into two pieces while the failure type-II fusion operation will inherit the entanglement of the standard cluster state by retaining the bonds attached to each other.

The significance of the scheme of writing quantum information into standard cluster state is three-fold. First, the scheme makes possible for us to substitute preferred data for unpreferred one from the cluster state of the one-way quantum computer. Take, for instance, the case that during the two-stage procedure a mistake in the initialization of the quantum state of the qubit a is found after the required cluster state $|\Phi\rangle_{C_N}$ has been already prepared. The initial quantum state of qubit a is $|\psi'_{in}\rangle_a = u|0\rangle + v|1\rangle$ rather than $|\psi_{in}\rangle = \alpha|0\rangle + \beta|1\rangle$. We can remove the qubit a from the cluster state by measuring the qubit a in the σ_z eigenbasis, and write the correct quantum state $|\psi_{in}\rangle$ prepared on the another qubit b into the standard cluster state $|\Psi\rangle_{C_{N-1}}$ by the type-I or type-II fusion gate. Second, the scheme makes it possible for us to prepare the quantum state of the one-way quantum into the standard cluster state $|\Psi\rangle_{C_N}$ before or when the quantum state of the data is provided. This maybe give a method to decrease the time consumption of the one-way quantum computation. Third, the scheme makes it possible for us to reuse the sub-cluster fragments, which are created by the failure fusion operation for the purpose of generating the cluster state of large size. These byproduct sub-clusters can be rearranged and used for some simple computation with putting quantum data into it by this writing-in scheme. It may provide a way to save the resource of the entanglement during quantum computation.

In this report we propose a scheme to write the quantum data into the one-way quantum computer by the type-I and type-II gates. It maybe give us some inspiration to make full use of the resources including time and entanglement.

Acknowledgement This work was financially supported by National Natural Science Foundation of China under Grant No. 10404007.

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